Path Integral Modeling of Stochastic Neuronal Dynamics

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ABSTRACT

We present investigations on stochastic dynamics of neuronal behavior in response to changes in ionic currents through the neuronal membrane. We work with the relative potential defined as the difference between the membrane potential and threshold firing potential of a neuron. Using a path integral representation, we obtain the conditional probability density of relative potential fluctuations as the fundamental solution of an appropriate Fokker-Planck equation. We do this for several forms of time-dependent current modulation coefficients which are defined in analogy to drift coefficients in the configuration space diffusion equation. In particular, we determine the conditional probability density for sinusoidal, exponential and polynomial time dependence of current modulation coefficients. To accommodate wider classes of behavior, we also obtain solutions for oscillatory but non-periodic time-dependence of current fluctuations modeled with special functions such as Bessel and Neumann functions. For interpretation of results, we show various plots of the conditional probability density as a function of relative membrane potential and time for the different types of ionic current modulation. Remarks are also made on possible connections with biophysical neuronal systems.